# Key for Take-Home Assignment 03

Bohr's model for the hydrogen atom predicts that the wavelength of the photon emitted when an electron moves from an initial shell, $n_i$, to a final shell, $n_f$, is

$$\frac{1}{\lambda} = R \times \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\}$$

where $n_f$ is smaller than $n_i$, and where $R$ has a value of $1.09737 \times 10^7 \text{ m}^{-1}$. We can extend this equation to any one-electron ion, such as $He^+$ or $Li^{2+}$, if we account for the charge on ion's nucleus, $Z$.

$$\frac{1}{\lambda} = R \times Z^2 \times \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\}$$

What is the wavelength, in nanometers ($1 \text{ nm} = 1.0 \times 10^{-9} \text{ m}$), of the photon that is emitted when the single electron in an ion of boron with a charge of +4 moves from the $n_i = 9$ shell to the $n_f = 5$ shell? What is the frequency of this photon in $s^{-1}$ and its energy in J? Note that the charge on the ion is not the same thing as the charge on ion's nucleus, $Z$.

**Solution**

Plugging in 5 for the charge on the nucleus (note that $Z$ is the charge on the nucleus, which is the atomic number, not the charge of the ion, which is $B^{4+}$ as there is just a single electron), 9 for the initial shell, and 5 for the final shell gives the wavelength as

$$\frac{1}{\lambda} = 1.09737 \times 10^7 \text{ m}^{-1} \times 5^2 \times \left\{ \frac{1}{5^2} - \frac{1}{9^2} \right\} = 7.59 \times 10^6 \text{ m}^{-1}$$

Taking the inverse gives the wavelength in meters as $1.32 \times 10^{-7}$ m or 132 nm.

To calculate the frequency, we note that $c = \lambda\nu$ where $c$ is the speed of light ($2.998 \times 10^8$ m/s); thus, we obtain a frequency of

$$\nu = \frac{c}{\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{1.32 \times 10^{-7} \text{ m}} = 2.27 \times 10^{15} \text{ s}^{-1}$$

To calculate the energy in Joules, we note that $E = \text{h}\nu$ where $h$ is Planck's cosntant ($6.626 \times 10^{-34}$ Js), or

$$E = h\nu = 6.626 \times 10^{-34} \text{ Js} \times 2.27 \times 10^{15} \text{ s}^{-1} = 1.51 \times 10^{-18} \text{ J}$$