Short Problem Set 3

For each of the following three problems

- explain, in one or two sentences, which significance test you will use
- state your null hypothesis, H_0 and your alternative hypothesis, H_A
- identify the degrees of freedom, ν
- report the test statistic's critical value
- calculate the test statistic's experimental value
- interpret the result of your statistical analysis

You may use R or a calculator to calculate means and standard deviations, but complete the remainder of the analysis by hand.

1. To evaluate a new employee, the manager of an environmental lab asks here to analyze a standard sample know to contain 0.520 ppb phenol. The analyst completes three analyses, obtaining an experimental mean of 0.513 ppb and a standard deviation of 0.0500 ppb. At an α of 0.05, is there any evidence of a systematic error in her work?

   This problem requires a two-tailed t–test of $\bar{x}$ to μ because we are comparing the analyst's experimetnal results to a known value for a standard sample and have no *a priori* reason to anticipate that a systematic error, if present, is in one direction only. The null hypothesis is $H_0 : \bar{x} = \mu$ and the alternative hypothesis is $H_A : \bar{x} \neq \mu$. The test statistic, t_{exp} is

   $$t_{exp} = \frac{(0.520 - 0.513)\sqrt(3)}{0.050} = 0.242$$

   and the critical two-tailed value for $t(0.05, 2)$ is 4.30 for $\alpha = 0.05$ and for two degrees of freedom. Because t_{exp} is less than $t(0.05, 2)$, we retain the null hypothesis and find no evidence that the difference between $\bar{x}$ and μ is due to anything other than uncertainty in the measurements at $\alpha = 0.05$.

2. The concentration of nitrate, NO_3^-, in public drinking water supplies is not allowed to exceed 50.0 ppm (as higher concentrations are linked to a condition called "blue baby syndrome" that affects newly-born infants). An analysis of four replicate samples drawn from a municipal water department gives the following results: 51.0 ppm, 51.3 ppm, 51.6 ppm, and 50.9 ppm. At an α of 0.05, is there any reason to suspect that the concentration of nitrate **exceeds** the legal limit? *Hmm... why is that word in a **bold** font?*

   This problem requires a one-tailed t–test of $\bar{x}$ to μ because we are interested in whether there is evidence that the concentration of nitrate exceeds a mandated standard. The null hypothesis is $H_0 : \bar{x} = \mu$ (we also could write this as $H_0 : \bar{x} \leq \mu$) and the alternative hypothesis is $H_A : \bar{x} > \mu$. The test statistic, t_{exp} is

   $$t_{exp} = \frac{(51.2 - 50.0)\sqrt(4)}{0.316} = 7.59$$

   and the critical one-tailed value for $t(0.05, 3)$ is 2.35 for $\alpha = 0.05$ and for three degrees of freedom. Because t_{exp} is greater than $t(0.05, 3)$, we reject the null hypothesis and find evidence that experimental uncertainty cannot explain the difference between $\bar{x}$ and μ at $\alpha = 0.05$.

3. The titanium content of steel is determined in two analytical labs by means of atomic absorption spectrometry. The first lab analyzes eight samples, obtaining results (in %w/w Ti) of 0.470, 0.448, 0.463, 0.449, 0.482, 0.454, 0.477, and 0.409. A second lab analyzes six samples, obtaining results of 0.529, 0.490, 0.489, 0.521, 0.486, and 0.502. At an α of 0.05, is there any evidence of a systematic difference between the results obtained by these two labs?

This problem requires a two-tailed t–test of $\bar{X}_1$ and $\bar{X}_1$ because we have no *a priori* reason to anticipate that one lab's results must be greater than that for the other lab. We know that the data are unpaired because the two labs analyze different numbers of samples. The mean and the standard deviation for the first lab are 0.456% and 0.0230%, respectively, and the mean and the standard deviation for the second lab are 0.503% and 0.0182%, respectively. The null hypothesis is $H_0 : \bar{x}_1 = barx_2$ and the alternative hypothesis is $H_A : \bar{x}_1 \neq barx_2$.

Before we complete the t–test, we first must determine if it is reasonable to pool the standard deviations, for which we use a two-tailed F–test using the following null hypothesis and altervative hypothesis: $H_0 : (s)_1^2 = (s)_2^2$ and $H_A : (s)_1^2 \neq (s)_2^2$. The test stastic, F_{exp}, is

$$F_{exp} = \frac{(0.0230)^2}{(0.0182)^2} = 1.59$$

and the critical two-tailed value for $F(0.05, 7, 5)$ is 6.853. Because $Fexp$ is less than $F(0.05, 7, 5)$, we retain the null hypothesis and have no reason at $\alpha = 0.05$ to believe that there is a signficiant difference between the variances and can pool the standard deviations; thus

$$s_{pooled} = \sqrt{\frac{(8-1)(0.0230)^2 + (6-1)(0.0182)^2}{8+6-2}} = 0.0211$$

The test statistic, t_{exp} is

$$t_{exp} = \frac{|0.503 - 0.456|}{0.0211} \times \sqrt{\frac{8 \times 6}{8+6}} = 4.06$$

and the critical two-tailed value for $t(0.05, 12)$ is 2.18 for $\alpha = 0.05$ and for 12 degrees of freedom. Because t_{exp} is greater than $t(0.05, 12)$, we reject the null hypothesis and find evidence that experimental uncertainty cannot explain the difference between the two labs at $\alpha = 0.05$.