

Short Problem Set 4

Due at the beginning of our next class. Be prepared to share your work in class.

1. Examine the comic from www.xkcd.com on the back of this page. Assuming that each test uses $\alpha = 0.05$, report the correct probability for a type I error.

Answer: With $k = 20$ comparisons, the probability of a type I error is

$$\alpha_{final} = 1 - (1 - \alpha_{test})^k = 1 - (1 - 0.05)^{20} = 0.6415$$

or 64%.

2. The following data set includes four replicate determinations of the amount of iron in three vitamin tablets (all results are mg Fe/100 g). Complete a one-way analysis of variance to determine if there is evidence of a systematic difference between the tablets at $\alpha = 0.05$. You may use your calculator, Excel, or R to calculate the global mean, the individual mean for each tablet, and the global variance, but complete all other calculations by hand.

trial	tablet a	tablet b	tablet c
1	5.67	5.75	4.74
2	5.67	5.47	4.45
3	5.55	5.43	4.65
4	5.57	5.45	4.94

Answer

$$\bar{\bar{x}} = 5.278$$

$$\bar{s}^2 = 0.2057$$

$$SS_{total} = (0.2057)(12 - 1) = 2.263$$

Individual means are 5.615 for tablet a, 5.525 for tablet b, and 4.695 for tablet c

$$SS_b = (4)(5.615 - 5.278)^2 + (4)(5.525 - 5.278)^2 + (4)(4.695 - 5.278)^2 = 2.058$$

$$SS_w = SS_{total} - SS_b = 2.263 - 2.058 = 0.205$$

$$s_b^2 = \frac{2.058}{2} = 1.029$$

$$s_w^2 = \frac{0.205}{9} = 0.0228$$

One-tailed F -Test

$$F_{exp} = \frac{1.029}{0.0228} = 45.1$$

$$F_{crit} = F(0.05, 2, 9) = 4.256$$

Because $F_{exp} > F_{crit}$ we find evidence at $\alpha = 0.05$ that the difference between the concentrations of iron in the three tablets cannot be explained by uncertainty in the analysis.

$$\sigma_{rand}^2 = s_w^2 = 0.0228$$

$$\sigma_{sys}^2 = \frac{s_b^2 - \sigma_{rand}^2}{\bar{n}} = 1.029 - 0.02284 = 0.252$$