Solution to Short Problem Set 3

1. This problem requires a two-tailed *t*-test of $\overline{X}$ to μ because we are comparing the analyte's experimental results to a known value from a standard sample and have no *a priori* reason to anticipate that a systematic error, if present, is in one direction only. The null hypothesis and alternative hypothesis are

   H_0: $\overline{X} = \mu$ and H_A: $\overline{X} \neq \mu$

   The test statistic, t_{exp}, is

   $$t_{\text{exp}} = \frac{(0.520 - 0.513)\sqrt{3}}{0.050} = 0.242$$

   and the critical value for $t(0.05, 2)$ is 4.30. Because $t_{exp} < t(0.05, 2)$, we find that uncertainty in the measurement process is sufficient to explain the difference between $\overline{X}$ and μ at α = 0.05.

2. This problem requires a one-tailed *t*-test of $\overline{X}$ to μ because we are interested in whether there is evidence that the concentration of nitrate exceeds a mandated standard. The null hypothesis and alternative hypothesis are

   H_0: $\overline{X} = \mu$ (or that H_0: $\overline{X} \leq \mu$) and H_A: $\overline{X} > \mu$

   The test statistic, t_{exp}, is

   $$t_{\text{exp}} = \frac{(51.2 - 50.0)\sqrt{4}}{0.316} = 7.59$$

   where the mean is 51.2 ppm and the standard deviation is 0.316 ppm. The critical value for $t(0.05, 3)$ is 2.35. Because $t_{exp} > t(0.05, 3)$, we find that uncertainty in the measurement process is not sufficient to explain the difference between $\overline{X}$ and μ at α = 0.05 and have reason to believe that the concentration of nitrate exceeds its limit.

3. This problem requires a two-tailed *t*-test of $\overline{X}_1$ and $\overline{X}_2$ for unpaired data. We know the test is two-tailed because we have no *a priori* expectation that the results for a particular lab must exceed the results for the other lab. We assume the data are unpaired because there is no evident relationship between the samples; that is, sample 1 for lab 1 presumably is different from sample 1 for lab 2. The mean and standard deviation for lab 1 are 0.456% and 0.0230%, respectively, and the mean and the standard deviation for lab 2 are 0.503% and 0.0182%, respectively. The null hypothesis and alternative hypothesis are

   H_0: $\overline{X}_1 = \overline{X}_2$ and H_A: $\overline{X}_1 \neq \overline{X}_2$

   Before we can complete the *t*-test, we first must determine if it is reasonable to pool the standard deviations, for which we use a two-tailed *F*-test with the following null and alternative hypotheses

   H_0: $(s_1)^2 = (s_2)^2$ and H_0: $(s_1)^2 \neq (s_2)^2$

   The test statistics, F_{exp}, is

   $$F_{\text{exp}} = \frac{(0.0230)^2}{(0.0182)^2} = 1.59$$

The critical value for $F(0.05, 7, 5)$ is 6.853. Because $F_{\text{exp}} < F(0.05, 7, 5)$, we have no reason to believe, at $\alpha = 0.05$, that there is a systematic difference in the standard deviations; thus, we pool the standard deviations, obtaining

$$s_{\text{pooled}} = \sqrt{\frac{(8-1)(0.0230)^2 + (6-1)(0.0182)^2}{8+6-2}} = 0.0211$$

The test statistic, t_{exp}, is

$$t_{\text{exp}} = \frac{0.503 - 0.456}{0.0211} \times \sqrt{\frac{8 \times 6}{8+6}} = 4.06$$

The critical value for $t(0.05, 12)$ is 2.18. Because $t_{\text{exp}} > t(0.05, 12)$, we find, at $\alpha = 0$, that there is reason to believe that the difference between the results of the two labs cannot be explained by uncertainty in the measurements.